

Request for help on a locus problem

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The following are email letters, with some small changes to allow readability, discussing a small locus problem that may be helpful for beginners in learning this topic.

(1) Email from S.L. (name initialized for confidentiality) dated 25/12/2013

Dear Professor,

I am S. L. from India and I have trying to avail of resources from the net on mathematics with a view to help my daughter who has taken mathematics in Grade XI. Recently both of us have been unable to solve a locus problem that was given to her in a test.

The problem is as defined below:

A is (-2,0) and P is any point on the curve $y^2 = 16x$. If Q bisects AP, find the equation of the locus of Q.

I would be grateful if you kindly throw light on how to solve the problem.

Thanks

S.L.

(2) My reply email on 26/12/2013

Dear S. L.,

The best way is to rewrite $y^2 = 16x$ in parametric form.

So you can write $P(t^2, 4t)$ as a point on $y^2 = 16x$, where t a parameter. The nice thing is that the curve becomes *a movable point*.

You may check that $x = t^2, y = 4t$ satisfies the equation $y^2 = 16x$.

The mid-point of AP is $M\left(\frac{t^2-2}{2}, \frac{4t+0}{2}\right)$.

So the locus of M is given by $x = \frac{t^2-2}{2}, y = 2t$. Hence $t = \frac{y}{2}$.

Subst. in x we get $x = \frac{\left(\frac{y}{2}\right)^2 - 2}{2} = \frac{y^2 - 8}{8}$.

Then you get the equation for M: $y^2 = 8x + 8$ which is a parabola.

BTW, I am not a professor, just a simple retired teacher of Queen's College, Hong Kong.

Hope the above calculation can solve your problem.

Best regards,

K.C.Yue

(3) Learning Tips:

Some parametric forms:

(a) Circle, $(x - a)^2 + (y - b)^2 = r^2$: $\begin{cases} x = a + r \cos t \\ y = b + r \sin t \end{cases}$, where $0 \leq t < 2\pi$

(b) Parabola, $y^2 = 4ax$: $\begin{cases} x = at^2 \\ y = 2at \end{cases}$, where $-\infty < t < \infty$

(c) Ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$: $\begin{cases} x = a \cos t \\ y = b \sin t \end{cases}$, where $0 \leq t < 2\pi$

(d) Hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$: $\begin{cases} x = a \sec t \\ y = b \tan t \end{cases}$, where $0 \leq t < 2\pi$

(4) My further email on 26/12/2013

Dear S. L.,

If you are not familiar with parametric form, you can let $P(a, b)$ be a point on $y^2 = 16x$ and let $M(x, y)$ be the mid point of PA . (Note that M is more important and we use (x, y) and P is less important and we use (a, b) . Both P and M are moving. We like to eliminate a, b later.)

Hence we have $x = \frac{a-2}{2}$, $y = \frac{b+0}{2}$

Then $a = 2x + 2$, $b = 2y$ (1)

But P is on $y^2 = 16x$, hence $b^2 = 16a$ (2)

Subst. (1) in (2), we get $(2y)^2 = 16(2x + 2)$

We finally get $y^2 = 8x + 8$

Bye for now,

K.C.Yue

(5) Email from S.L. dated 26/12/2013

Thank you Sir for the solution. You have made it look very simple.

I am not familiar with the parametric form but I am conversant with your second method.

Thanks again for the great help.

Regards

S.L.